

ASSIGNMENT 1

POINTS: 35

DATE GIVEN: 08-MAR-2021

DUE: 20-MAR-2021

Rules:

- You are strongly encouraged to work *independently*. That is the best way to understand the subject.
- Write the solutions on your own and honorably *acknowledge* the sources if any. <http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html>
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proofs done in the class.
- Submit your solutions, before time, to your TA. Preferably, give the TA a printed/pdf copy of your LaTeXed or Word processed solution sheet.

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Question 1: [5 points] Two dice (with six faces) are rolled and their scores are S_0, S_1 . What is the probability that the quadratic equation $x^2 + S_1x + S_0 = 0$ has *real* roots?

Question 2: [3 points] An urn contains n Red and n Blue balls. A fair die with n sides is rolled; if r -th face appears then r balls are removed from the urn and placed in a bag. Now pick a random ball from the bag. What is the probability that it is a Red ball?

Question 3: [6 points] [Murphy's Law] A fair coin is tossed repeatedly. Let s denote any fixed sequence of heads and tails of length r . Show that with probability *one* the sequence s will *eventually* appear in n tosses of the coin (as $n \rightarrow \infty$).

Thus, anything that can go wrong, will go wrong!

Question 4: [9 points] There are six letters and envelopes each. They are equally distributed in three colors: R, W and B. Place the letters randomly in envelopes arranged in a line. What is the probability that no letter is in an envelope of the same color?

Question 5: [7+5 points] [Recruitment] The famous company BigBucks wants to visit IITK to hire the best qualified student. There are n applicants. The constraints put by the Placements Office is:

- (1) The interview panel will select only *one* student.
- (2) Each applicant has to be informed about the accept/reject decision *immediately* after the interview. The quality of the candidate, revealed by the interview, can be assumed to be a real number in $[0, 1]$.
- (3) The applicants appear for interview in a uniformly *random* order.

Devise an algorithm for BigBucks' hiring. Give an analysis to estimate the probability that BigBucks ends up hiring *the* best applicant in IITK.

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