CS203B - PROBABILITY FOR COMPUTER SCIENCE

NITIN SAXENA

ASSIGNMENT 2

POINTS: 35

DATE GIVEN: 25-MAR-2021 DUE: 09-APR-2021

Rules:

- You are strongly encouraged to work *independently*. That is the best way to understand the subject.
- Write the solutions on your own and honorably *acknowledge* the sources if any. `cse.iitk.ac.in/pages/AntiCheatingPolicy.html`
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proofs done in the class.
- Submit your solutions, before time, to your TA via MooKit. Preferably, submit a printed/pdf copy of your LaTeXed or Word processed solution sheet.
  `170574--190108: pbisht@iitk.ac.in`
  `190117--228: subinp@iitk.ac.in`
  `190229--381: debojyot@iitk.ac.in`
  `190395--553: bhargav@iitk.ac.in`
  `190562--810: sayak@iitk.ac.in`
  `190824--191181: akhis@iitk.ac.in`

Question 1: *[5 points]* We toss a fair coin n times. Prove that the length of the longest sequence of consecutive Heads will not be more than $2\log n$ with probability at least $1 - 1/n$.

Question 2: *[5+3 points]* There are two bridges from town A to town B and two bridges from town B to town C. Each of the four bridges is blocked by snow with probability p, independent of the others.

- Find the probability that there is an open bridge from A to B given that there is no open route from A to C.

- Say, in addition, there is a direct bridge from A to C; this bridge being blocked with probability p independently of the others. Find the required conditional probability mentioned above.

Question 3: *[7 points]* In a male-dominated sexist society, there are n married couples. Each married couple aims at having at least one male child. So each couple practises the following rule: Keep on producing children until either the number of children becomes 10 or one male child is born. What will be the ratio of males and females in the next generation? Assume that each child is going to be male or female with equal probability. Give mathematical reason in support of your answer.

Question 4: *[5 points]* X is a continuous random variable with probability distribution function $F_X(x) := P(X \leq x)$. Y is another continuous random variable with probability distribution function $F_Y(y) := P(Y \leq y)$. Variables X and Y are independent. Let $Z := \min(X, Y)$. What is the probability distribution function F_Z for Z?

Question 5: *[10 points]* Let $Q : [n] \to [n]$ be a bijection. There is a standard technique to decompose Q as a *product of disjoint cycles*. For example, $C_1 := (i_1 := 1, Q(i_1), Q^2(i_1), \ldots)$ gives the first cycle. Next, consider an element $i_2 \notin C_1$ to define $C_2 := (i_2, Q(i_2), Q^2(i_2), \ldots)$ as the second cycle; and so on till all the elements $[n]$ are covered. Then, $(C_1, C_2, \ldots, C_r)$ is essentially a *unique* cycle decomposition for Q.

Pick a *random* bijection Q on $[n]$. What is the expected number of cycles r for Q?

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